$Q_c(\alpha, \beta)$ does not contain p^2, as we have just seen. But these two numbers contain no other factor in common, as is seen from Theorem I. Hence $Q_c(\alpha, \beta)$ contains a prime not in $\alpha^{c/p}-\beta^{c/p}$ at least in every case for which $Q_c(\alpha, \beta)\neq p$ and also in every case for which $\alpha^{c/p}-\beta^{c/p}$ is not divisible by p, unless $Q_c(\alpha, \beta)=1$.

COROLLARY. $\alpha^c-\beta^c$ *has always a prime factor not dividing* $\alpha^s-\beta^s$ $(s<c)$ *except in the cases mentioned in Theorem VII.*

NOTE ON THE EXTENSION OF THE EXPONENTIAL THEOREM.

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In the writer's paper in the June-July number of the MONTHLY, pp. 101-106, it will be observed that the complex roots, infinite in number, arising from the application of an incommensurable exponent are tacitly neglected, only the single real root being used. In fact this is the only root that is of much practical value. It is the only root that is usually considered in the extension of the binomial theorem for the expansion of $(1+x)^n$, even for a commensurable fractional exponent. The finite number of complex roots can be easily expressed if wanted. But when the exponent is incommensurable the number of complex roots becomes infinite and the complex roots become indeterminate.

In both cases, vis., of the binomial theorem, and the exponential theorem, which depends on the binomial theorem, the developments already obtained would have to be multiplied by $\cos\varphi+i\sin\varphi$, to obtain the complete development, where φ admits the value of zero, and has besides an infinite number of values which are indeterminate, when the exponent is incommensurable. All the values would have the same modulus.

These facts are so obvious that it will doubtless appear superfluous to many readers to call attention to them; yet for others it may not be amiss to do so.